

SECTION 10

MEDIA AND ANTENNA CORRECTIONS

Contents

10.1	Introduction	10-4
10.2	Media Corrections in the Regres Editor	10-5
10.2.1	Individual-Leg Troposphere Corrections.....	10-6
10.2.1.1	Introduction	10-6
10.2.1.2	Tropospheric Zenith Dry and Wet Range Corrections	10-7
10.2.1.3	Mapping Functions	10-8
10.2.1.3.1	Chao Model.....	10-9
10.2.1.3.2	Niell Model.....	10-9
10.2.1.4	Partial Derivatives.....	10-11
10.2.2	Individual-Leg Charged-Particle Corrections.....	10-12
10.2.3	Light-Time Corrections.....	10-13
10.2.3.1	Definitions of Precision Light Times.....	10-14
10.2.3.1.1	One-Way Spacecraft Data Types.....	10-14
10.2.3.1.2	Round-Trip Spacecraft Data Types.....	10-16
10.2.3.1.3	Quasar Interferometry Data Types.....	10-17

SECTION 10

10.2.3.2	Corrections to Precision Light Times.....	10–18
10.2.3.2.1	One-Way Spacecraft Data Types.....	10–18
10.2.3.2.2	Round-Trip Spacecraft Data Types.....	10–20
10.2.3.2.3	Quasar Interferometry Data Types.....	10–21
10.2.3.3	Time Arguments for Media Corrections.....	10–22
10.2.3.3.1	Spacecraft Reception Times, and Quasar Reception Times at Receiving Station 1.....	10–22
10.2.3.3.2	Spacecraft Transmission Times, and Quasar Reception Times at Receiving Station 2.....	10–24
10.3	Ionosphere Partials Model.....	10–25
10.3.1	Ionosphere Model and Individual-Leg Partial Derivatives	10–26
10.3.2	Partial Derivatives of Precision Light Times	10–29
10.4	Solar Corona Model.....	10–30
10.4.1	Calculation of Arguments for Solar Corona Corrections.....	10–31
10.4.2	Individual-Leg Solar Corona Corrections.....	10–33
10.4.3	Adding Solar Corona Corrections to the Light- Time Solutions	10–36
10.4.4	Individual-Leg Solar Corona Partial Derivatives	10–37

MEDIA AND ANTENNA CORRECTIONS

10.4.5	Partial Derivatives of Precision Light Times	10-38
10.5	Antenna Corrections	10-39
10.5.1	Introduction	10-39
10.5.2	Antenna Types and Corrections	10-42

Figure

10-1	Antenna Correction	10-41
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Table

10-1	Antenna Types	10-43
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10.1 INTRODUCTION

This section describes how media corrections to the computed values of observables are calculated in the Regres editor.¹ Media corrections consist of corrections due to the Earth's troposphere and corrections due to charged particles. The charged particles can be in the Earth's ionosphere, in space (space plasma), or in the solar corona. This section also gives the formulation for calculating antenna corrections for the computed values of observables in program Regres. These corrections are non-zero if the axes of the antenna at a DSN tracking station on Earth do not intersect. Partial derivatives are given for two parameters of the troposphere, two parameters of the ionosphere, and three parameters of the solar corona.

Troposphere corrections and all charged-particle corrections except those calculated from the solar corona model are calculated in the Regres editor. These calculations are described in Section 10.2. Sections 10.2.1 and 10.2.2 describe the troposphere and charged-particle corrections, respectively. The contributions of the individual-leg troposphere and charged-particle corrections to the calculated precision round-trip light time ρ , the one-way light time ρ_1 , and the quasar delay τ are given in Section 10.2.3. These light-time corrections are used in equations given in Section 13 to calculate additive corrections to the computed values of observables due to the troposphere and charged particles.

The ionosphere partials model is given in Section 10.3. This model is used to derive equations for partial derivatives with respect to two solve-for parameters of the ionosphere. Solved-for corrections to these two parameters should be considered to be corrections to the charged-particle corrections calculated in the Regres editor. Since the formulation for calculating computed values of observables does not include an ionosphere model, computed observables cannot be corrected for the changes in the solved-for ionosphere

¹The Regres editor is the Editor Library, which is included in programs Regres and Edit. Hence, the user has the option of performing data editing and calculating media corrections in program Regres or in program Edit, which is executed after program Regres.

parameters. That is, you cannot do an iterative solution for the two ionosphere parameters.

The solar corona model, which is contained in program Regres, is described in Section 10.4. This model is included in the formulation for the computed values of observables, and iterative solutions for the three solve-for parameters of this model can be obtained.

Section 10.5 gives the formulation for calculating antenna corrections for the computed values of observables.

10.2 MEDIA CORRECTIONS IN THE REGRES EDITOR

The Regres editor calculates media (*i.e.*, troposphere and charged particle) corrections to the computed values of observables, miscellaneous user-specified corrections to the computed values of observables, weights for the observables, and performs data editing (*e.g.*, deleting specified data points). It can also add noise to the computed values of observables. The inputs to the Regres editor for performing these functions are the so-called CSP (command statement processor) commands, which are contained in the CSP file. The “English” version of the CSP file is converted to the computer language version by program Translate.

The $(O - C)$ residual is the observed value of an observable minus the computed value. The computed value C is the value calculated in program Regres and does not include any corrections calculated in the Regres editor. The $(O - C)$ residual is placed in the variable RESID on the Regres file. The sum of the corrections δC to the computed value C of the observable calculated in the Regres editor is placed in the variable CRESID on the Regres file. Programs downstream of Regres and Edit (if the Regres editor is executed in program Edit instead of in program Regres) can calculate the corrected residual $[O - (C + \delta C)]$ as RESID – CRESID.

Individual-leg corrections due to the troposphere and charged particles are calculated as described in Subsections 10.2.1 and 10.2.2, respectively.

SECTION 10

Corrections to the round-trip and one-way light times and the quasar delay are calculated from sums and differences of individual-leg corrections in Subsection 10.2.3. These light-time corrections are used in Section 13 to calculate corrections to the computed values of observables. These equations are evaluated in the Regres editor.

10.2.1 INDIVIDUAL-LEG TROPOSPHERE CORRECTIONS

10.2.1.1 Introduction

For the up-leg light path from a tracking station on Earth to a spacecraft or the down-leg light path from a spacecraft or a quasar to a tracking station on Earth, the increase in the light time due to the Earth's troposphere is the tropospheric range correction $\Delta_T \rho$ (evaluated at the reception or transmission time at the tracking station on Earth) in meters divided by $10^3 c$, where c is the speed of light in kilometers per second. The tropospheric range correction $\Delta_T \rho$ is calculated from:

$$\Delta_T \rho = \rho_{z_{\text{dry}}} R_{\text{dry}}(\gamma) + \rho_{z_{\text{wet}}} R_{\text{wet}}(\gamma) \quad \text{m} \quad (10-1)$$

where $\rho_{z_{\text{dry}}}$ and $\rho_{z_{\text{wet}}}$ are tropospheric zenith range corrections in meters due to the dry and wet components of the troposphere. The functions $R_{\text{dry}}(\gamma)$ and $R_{\text{wet}}(\gamma)$ map the zenith range corrections to the elevation angle γ of the light path at the transmission time or reception time at the tracking station on Earth. The elevation angle γ is specifically the unrefracted auxiliary elevation angle calculated as described in Section 9.3.

Section 10.2.1.2 describes the calculation of the tropospheric zenith dry and wet range corrections. Each of these corrections is the sum of a correction calculated from a seasonal model in the Regres editor plus a constant solve-for correction obtained using partial derivatives given in Section 10.2.1.4. Calculation of the mapping functions is described in Section 10.2.1.3

10.2.1.2 Tropospheric Zenith Dry and Wet Range Corrections

The seasonal model for the tropospheric zenith dry and wet range corrections represents these quantities as normalized power series or Fourier series. The current model is based upon the original work of Chao (1971). The current model was obtained as described on pages 3 to 9 of Estefan and Sovers (1994). At each of the three DSN complexes, measured values of the following quantities were obtained: the surface pressure, the surface temperature, the surface relative humidity, and the temperature lapse rate (the altitude temperature gradient). Monthly averages of these four parameters were calculated over a two-calendar-year period. This data was used to calculate the tropospheric zenith dry and wet range corrections from Eqs. (1) and (2) of Estefan and Sovers (1994), where these equations were obtained from Berman (1970). The calculated tropospheric zenith dry and wet range corrections were fit with normalized power series and also with Fourier series. The tropospheric zenith dry and wet range corrections can be calculated from normalized power series using Eq. (3) of Estefan and Sovers (1994). The coefficients in this equation are contained in the CSP commands given in Figure 3a on page 8 of this reference. The data in these commands can be applied to any two-calendar-year timespan by changing the two-digit year in the FROM and TO times. The tropospheric zenith dry and wet range corrections can be calculated from Fourier series using Eq. (4) of Estefan and Sovers (1994). The coefficients in this equation are contained in the CSP commands given in Figure 3b on page 9 of this reference. The data in these commands applies for any time. According to Estefan and Sovers (1994), better fits to tracking data are obtained when representing the zenith tropospheric corrections as Fourier series. The time arguments needed to evaluate the normalized power series or the Fourier series are specified in Section 10.2.3.3.

Each of the two sets of CSP commands referred to above contain coefficients for calculating tropospheric zenith dry and wet range corrections at the DSN complexes at Goldstone, Canberra, and Madrid. The dry and wet corrections calculated at each complex apply for each tracking station at the complex. Additional CSP commands would be required to calculate corrections

SECTION 10

at an isolated tracking station. In addition to the modelled tropospheric zenith range corrections calculated in the Regres editor, the user can estimate constant corrections to the tropospheric zenith dry and wet range corrections at each DSN complex or isolated tracking station. These solve-for corrections are obtained using the partial derivatives given in Section 10.2.1.4. The total tropospheric zenith dry and wet range corrections used in Eq. (10–1) for all tracking stations at a DSN complex or at an isolated tracking station are thus given by:

$$\rho_{z_{\text{dry}}} = \left(\rho_{z_{\text{dry}}} \right)_{\text{model}} + \Delta \rho_{z_{\text{dry}}} \quad \text{m} \quad (10-2)$$

$$\rho_{z_{\text{wet}}} = \left(\rho_{z_{\text{wet}}} \right)_{\text{model}} + \Delta \rho_{z_{\text{wet}}} \quad \text{m} \quad (10-3)$$

where the first terms are the modelled corrections calculated in the Regres editor and the second terms are the solved-for constant corrections obtained using the partial derivatives given in Section 10.2.1.4.

The DSN has the capability of calculating corrections to the seasonal model for the tropospheric zenith dry and wet range corrections, where the corrections are obtained from real-time measurements of atmospheric parameters. These corrections can be represented as normalized power series or Fourier series and can be included in the CSP file. The Regres editor will evaluate these corrections and add them to the seasonal model. Calculation of the corrections is described in Section 3.1 of Estefan and Sovers (1994).

10.2.1.3 Mapping Functions

This section describes the calculation of the mapping functions $R_{\text{dry}}(\gamma)$ and $R_{\text{wet}}(\gamma)$, which are used in Eq. (10–1). The user can calculate the mapping functions from the original Chao model, which is described in Subsection 10.2.1.3.1 or from the newer Niell model, which is described in Subsection 10.2.1.3.2.

10.2.1.3.1 Chao Model

The mapping functions $R_{\text{dry}}(\gamma)$ and $R_{\text{wet}}(\gamma)$ are evaluated by interpolating Chao's dry (TABDRY) and wet (TABWET) mapping tables with the unrefracted elevation angle γ . These tables contain the values of the mapping function every 0.1° from 0° to 10° and every 0.5° from 10° to 90° . These tables are given in the Appendix to Estefan and Sovers (1994). Eq. (6) of this reference is the interpolation formula.

The development of Chao's mapping tables is discussed in Section 2.2 of Estefan and Sovers (1994) and in Mottinger (1984).

10.2.1.3.2 Niell Model

The mapping functions $R_{\text{dry}}(\gamma)$ and $R_{\text{wet}}(\gamma)$ are calculated from Eqs. (50) to (56) and Tables 4a and 4b in Section 4.5 of Estefan and Sovers (1994). This section will give a few corrections and additions to this formulation and will describe how the required inputs to this model are calculated. The corrections and additions were obtained by comparing Section 4.5 of the reference to the actual code obtained from Arthur Niell and from a discussion with Arthur Niell.

In Eq. (52) of Estefan and Sovers (1994), the sign of the second term must be changed from positive to negative. The second term contains the cosine of an argument. If the geodetic latitude ϕ_g of the tracking station on Earth is positive, the cosine function should be:

$$\cos \left[2\pi \frac{t-28}{365.25} \right] \quad (10-4)$$

In Eq. (52) of the reference, the 2π factor was omitted. If the geodetic latitude ϕ_g of the tracking station is negative, the cosine function (10-4) must be replaced with:

$$\cos \left[2\pi \left(\frac{t-28}{365.25} + \frac{1}{2} \right) \right] \quad (10-5)$$

SECTION 10

Eq. (10–5) was omitted in the reference. In Eqs. (10–4) and (10–5), t is time past the last January 0, 0^h in days of Coordinated Universal Time UTC or station time ST.

The coefficients in Eqs. (52) and (56) of Estefan and Sovers (1994) are obtained from Tables 4a and 4b of that reference as a function of the absolute value of the geodetic latitude. The tables contain the values of the coefficients at geodetic latitudes of 15°, 30°, 45°, 60°, and 75°. For geodetic latitudes between 15° and 75°, the coefficients are obtained by linear interpolation. For absolute geodetic latitudes less than 15°, the values of the coefficients at 15° are used. For absolute geodetic latitudes greater than 75°, the values of the coefficients at 75° are used.

The Niell mapping function was obtained from Niell (1996). The mapping function is given by Eq. (4). However, the numerator of this equation is one divided by the correct numerator, and the denominator of this equation is one divided by the correct denominator. Hence, in order to obtain the correct mapping function from Eq. (4), the “one divided by” in the numerator and the “one divided by” in the denominator must be removed. In Eq. (5) of Niell (1996), the sign of the second term must be changed from positive to negative.

One of the inputs to this model is the geodetic latitude ϕ_g of the tracking station. Given the geocentric latitude ϕ and the geocentric radius r of the tracking station referred to the mean pole, prime meridian, and equator of 1903.0, the geodetic latitude ϕ_g can be calculated to sufficient accuracy from Eqs. (9–6) to (9–8).

Another input to this model is the height of the tracking station above mean sea level (the geoid), which according to Arthur Niell can be approximated with the height above the reference ellipsoid. Given the spin radius u of the tracking station (measured from the 1903.0 pole to the tracking station), the geodetic latitude ϕ_g calculated as described above, and the ellipsoid parameters listed after Eq. (9–7), the height h above the reference ellipsoid can be calculated from:

$$h = \frac{u}{\cos \phi_g} - \frac{a_e}{\sqrt{1 - e^2 \sin^2 \phi_g}} \quad \text{km} \quad (10-6)$$

This is the same as Eqs. (5.53) and (5.54) of Sovers and Jacobs (1996).

The time t (in the UTC or ST time scales) in days past the last January 0, 0^h can be obtained as follows. Time in the ODP (in the ET, TAI, UTC, UT1, and ST time scales) is represented as seconds past J2000. The reception time or transmission time at a tracking station on Earth (in UTC or ST) is converted to a calendar date. The calendar date is then converted to days past January 0, 0^h.

In addition to these inputs, the primary input required to compute the dry and wet mapping functions is the unrefracted auxiliary elevation angle γ , which is calculated as described in Section 9.3.

10.2.1.4 Partial Derivatives

From Eqs. (10-1) to (10-3), the partial derivatives of the tropospheric range correction with respect to the solve-for constant corrections to the modelled tropospheric zenith dry and wet range corrections at each DSN complex or isolated tracking station are given by the corresponding dry and wet mapping functions:

$$\frac{\partial \Delta_T \rho}{\partial \Delta \rho_{z \text{ dry}}} = R_{\text{dry}}(\gamma) \quad \text{dry} \rightarrow \text{wet} \quad (10-7)$$

If the user has selected the Chao mapping functions, these partial derivatives are evaluated from the following approximations to Chao's mapping tables, which were obtained from Eq. (19) on page 75 of Chao (1974):

$$R_{\text{dry}}(\gamma) = \frac{1}{\sin \gamma + \frac{A_{\text{dry}}}{\tan \gamma + B_{\text{dry}}}} \quad \text{dry} \rightarrow \text{wet} \quad (10-8)$$

where

SECTION 10

$$\begin{aligned} A_{\text{dry}} &= 0.00143 \\ B_{\text{dry}} &= 0.0445 \end{aligned} \tag{10-9}$$

and

$$\begin{aligned} A_{\text{wet}} &= 0.00035 \\ B_{\text{wet}} &= 0.017 \end{aligned} \tag{10-10}$$

From Chao (1974), these approximate mapping functions are in error by less than 1% for elevation angles greater than 1 degree.

If the user has selected the Niell mapping functions, the dry and wet mapping functions in Eqs. (10-7) are calculated from the formulation specified in Section 10.2.1.3.2.

10.2.2 INDIVIDUAL-LEG CHARGED-PARTICLE CORRECTIONS

For the up-leg light path from a tracking station on Earth to a spacecraft or the down-leg light path from a spacecraft or a quasar to a tracking station on Earth, the change in the light time due to charged particles along the light path is the charged-particle range correction $\Delta_{\text{CP}}\rho$ (evaluated at the reception time or transmission time at the tracking station on Earth) in meters divided by 10^3c , where c is the speed of light in kilometers per second. The charged-particle range correction $\Delta_{\text{CP}}\rho$ is calculated from:

$$\Delta_{\text{CP}}\rho = \pm \delta_{\text{CP}}\rho \left(\frac{2295 \times 10^6 \text{ Hz}}{f} \right)^2 \quad \text{m} \tag{10-11}$$

where $\delta_{\text{CP}}\rho$ is the charged-particle range correction in meters at the standard frequency of 2295×10^6 Hz, and f is the transmitter frequency in Hz for the specific leg of the light path.

The charged-particle range correction $\delta_{\text{CP}}\rho$ along the light path between any tracking station at a specific DSN complex or a specific tracking station on

Earth and a specific spacecraft or a specific quasar is calculated in the Regres editor as a normalized power series, a Fourier series, or a constant. These corrections are calculated the same way as tropospheric zenith dry and wet range corrections, as described in Section 10.2.1.2. Also, the input CSP commands for calculating charged-particle range corrections have the same general format as those used for tropospheric zenith dry and wet range corrections.

The algorithm for calculating the transmitter frequency f for each leg of each light path is given in Section 13.2.8.

The sign of the charged-particle range correction $\Delta_{\text{CP}\rho}$ given by Eq. (10–11) is negative for doppler and narrowband spacecraft or quasar interferometry data types and positive for range and wideband spacecraft or quasar interferometry data types.

Most of the input CSP commands for calculating the charged-particle range corrections represent the effects of the charged particles of the Earth's ionosphere and are derived by processing dual-frequency GPS data (transmitted from a GPS satellite to a GPS receiving station on Earth). The charged-particle range corrections along the directions to several GPS satellites can be interpolated to give the charged-particle range corrections along the light path to a specific spacecraft or quasar.

10.2.3 LIGHT-TIME CORRECTIONS

Computed values of doppler, range, and spacecraft or quasar interferometry data types are calculated from one, two, or four computed precision values of the one-way light time ρ_1 , the round-trip light time ρ , or the quasar delay τ . The round-trip light time is two-way if the transmitting station on Earth is also the receiving station. If the receiving station on Earth is not the transmitting station, the round-trip light time is three-way. Subsection 10.2.3.1 gives the definitions of the three precision light times: ρ_1 , ρ , and τ . The formulations for calculating these precision light times are given in Section 11. Subsection 10.2.3.2 gives equations for calculating media corrections to the three precision light times. These corrections are calculated from sums and differences

SECTION 10

of individual-leg tropospheric and charged-particle corrections in the Regres editor. They are used to calculate media corrections to the computed values of observables in the Regres editor. The equations for calculating media corrections to the computed observables from media corrections to the computed precision light times are given in Section 13. The media corrections and other user-specified corrections to the computed observables are placed on the Regres file in the variable CRESID, as discussed in Section 10.2.

In order to calculate media corrections to the three precision light times, the Regres editor needs a variety of reception times and transmission times. These epochs must be calculated in the Regres editor from quantities on the Regres file. The equations for performing these calculations are given in Subsection 10.2.3.3.

In Section 10, the receiving station can be a receiving station on Earth or a receiving Earth satellite. Similarly, the transmitting station can be a transmitting station on Earth or a transmitting Earth satellite.

10.2.3.1 Definitions of Precision Light Times

10.2.3.1.1 One-Way Spacecraft Data Types

The precision one-way light time, used to calculate the computed values of one-way doppler (F_1) observables and one-way wideband (IWS) and narrowband (INS) spacecraft interferometry observables, is defined to be:

$$\rho_1 = t_3(ST) - t_2(ET) \quad \text{s} \quad (10-12)$$

where $t_3(ST)$ is the reception time in station time ST at the receiving station and $t_2(ET)$ is the transmission time in coordinate time ET at the spacecraft.

Computed values of F_1 observables are calculated from the precision one-way light time ρ_{1_e} at the end of the doppler count interval T_c minus the precision one-way light time ρ_{1_s} at the start of the count interval:

MEDIA AND ANTENNA CORRECTIONS

$$\rho_{1_e} = t_{3_e}(\text{ST}) - t_{2_e}(\text{ET}) \quad \text{s} \quad (10-13)$$

$$\rho_{1_s} = t_{3_s}(\text{ST}) - t_{2_s}(\text{ET}) \quad \text{s} \quad (10-14)$$

where $t_{3_e}(\text{ST})$ and $t_{3_s}(\text{ST})$ are reception times at the receiving station at the end and start of the doppler count interval T_c . The epochs $t_{2_e}(\text{ET})$ and $t_{2_s}(\text{ET})$ are the corresponding transmission times at the spacecraft. Note that the count interval T_c is equal to:

$$T_c = t_{3_e}(\text{ST}) - t_{3_s}(\text{ST}) \quad \text{s} \quad (10-15)$$

The count interval T_c is an integer (*e.g.*, 60 s or 600 s).

The computed value of a one-way wideband spacecraft interferometry (*IWS*) observable is calculated from ρ_1 defined by Eq. (10-12) at receiving station 2 minus ρ_1 at receiving station 1 at the common reception time $t_3(\text{ST})$.

The computed value of a one-way narrowband spacecraft interferometry (*INS*) observable is calculated from F_1 at receiving station 2 minus F_1 at receiving station 1, where the two computed F_1 observables have the same values of $t_{3_e}(\text{ST})$, $t_{3_s}(\text{ST})$, and T_c . Each of the two computed F_1 observables is calculated from the difference of the two precision one-way light times defined by Eqs. (10-13) and (10-14).

The precision one-way light time, used to calculate the computed values of GPS/TOPEX pseudo-range and carrier-phase observables, is defined to be:

$$\rho_1 = t_3(\text{ST}) - t_2(\text{ST}) \quad \text{s} \quad (10-16)$$

where $t_3(\text{ST})$ is the reception time in station time ST at the receiving TOPEX satellite or GPS receiving station on Earth and $t_2(\text{ST})$ is the transmission time at the GPS satellite in station time ST.

SECTION 10

10.2.3.1.2 Round-Trip Spacecraft Data Types

The precision round-trip light time, used to calculate the computed values of two-way and three-way range (ρ_2 and ρ_3) observables, two-way and three-way doppler (F_2 and F_3) observables, and round-trip wideband (*IWS*) and narrowband (*INS*) spacecraft interferometry observables, is defined to be:

$$\rho = t_3(\text{ST}) - t_1(\text{ST}) \quad \text{s} \quad (10-17)$$

where $t_3(\text{ST})$ is the reception time in station time ST at the receiving station and $t_1(\text{ST})$ is the transmission time in station time ST at the transmitting station. If the transmitting station is the receiving station, the precision round-trip light time is two-way. Otherwise, it is three-way.

Computed values of two-way range (ρ_2) and three-way range (ρ_3) observables are calculated from the precision round-trip light time ρ defined by Eq. (10-17).

Computed values of two-way doppler (F_2) and three-way doppler (F_3) observables are calculated from the precision round-trip light time ρ_e at the end of the doppler count interval T_c and the precision round-trip light time ρ_s at the start of the count interval:

$$\rho_e = t_{3_e}(\text{ST}) - t_{1_e}(\text{ST}) \quad \text{s} \quad (10-18)$$

$$\rho_s = t_{3_s}(\text{ST}) - t_{1_s}(\text{ST}) \quad \text{s} \quad (10-19)$$

where $t_{3_e}(\text{ST})$ and $t_{3_s}(\text{ST})$ are reception times at the receiving station at the end and start of the doppler count interval T_c . The epochs $t_{1_e}(\text{ST})$ and $t_{1_s}(\text{ST})$ are the corresponding transmission times at the transmitting station.

The computed value of a round-trip wideband spacecraft interferometry (*IWS*) observable is calculated from ρ defined by Eq. (10-17) at receiving station 2 minus ρ at receiving station 1 at the common reception time $t_3(\text{ST})$.

The computed value of a round-trip narrowband spacecraft interferometry (*INS*) observable is calculated from F_2 or F_3 at receiving station 2 minus F_2 or F_3 at receiving station 1, where the two computed doppler observables have the same values of $t_{3_e}(\text{ST})$, $t_{3_s}(\text{ST})$, and T_c . Each of the two computed doppler observables is calculated from the two precision round-trip light times defined by Eqs. (10–18) and (10–19).

10.2.3.1.3 Quasar Interferometry Data Types

The precision quasar delay τ , used to calculate the computed values of wideband (*IWQ*) and narrowband (*INQ*) quasar interferometry observables, is defined to be:

$$\tau = t_2(\text{ST}) - t_1(\text{ST}) \quad \text{s} \quad (10-20)$$

where $t_2(\text{ST})$ and $t_1(\text{ST})$ are the reception times of the quasar wavefront at receiving stations 2 and 1, respectively, in station time ST.

The computed value of a wideband quasar interferometry (*IWQ*) observable is calculated from the precision quasar delay τ defined by Eq. (10–20).

The computed value of a narrowband quasar interferometry (*INQ*) observable is calculated from the precision quasar delay τ_e at the end of the count interval T_c minus the precision quasar delay τ_s at the start of the count interval:

$$\tau_e = t_{2_e}(\text{ST}) - t_{1_e}(\text{ST}) \quad \text{s} \quad (10-21)$$

$$\tau_s = t_{2_s}(\text{ST}) - t_{1_s}(\text{ST}) \quad \text{s} \quad (10-22)$$

where $t_{1_e}(\text{ST})$ and $t_{1_s}(\text{ST})$ are reception times at receiving station 1 at the end and start of the count interval T_c . The epochs $t_{2_e}(\text{ST})$ and $t_{2_s}(\text{ST})$ are the corresponding reception times at receiving station 2. Note that the count interval T_c is equal to:

SECTION 10

$$T_c = t_{1_e}(\text{ST}) - t_{1_s}(\text{ST}) \quad \text{s} \quad (10-23)$$

10.2.3.2 Corrections to Precision Light Times

Section 10.2.3.1 defined the three types of precision light times that are computed in program Regres and identified which of these precision light times are computed in calculating the computed value of each data type. This section will give equations for corrections to each of these computed precision light times due to the troposphere and charged particles. These corrections are input to equations given in Section 13 to give the media corrections to computed values of observables. These equations are evaluated in the Regres editor.

In the equations given in the following three subsections, the tropospheric range correction $\Delta_T \rho(t_i)$ in meters at the reception time or transmission time t_i is calculated from Eqs. (10-1) to (10-3) as described in Section 10.2.1. The corresponding charged-particle range correction $\Delta_{CP} \rho(t_i)$ is calculated from Eq. (10-11) as described in Section 10.2.2. If a transmitting station or a receiving station is an Earth satellite, the indicated tropospheric range correction is zero and the charged-particle correction does not include the effects of the Earth's ionosphere. Unless inputs are available for space plasma or charged particles of the solar corona, the charged-particle range correction will be zero.

10.2.3.2.1 One-Way Spacecraft Data Types

For the computed values of one-way doppler (F_1) observables, the media corrections to the precision one-way light times calculated at the end and start of the doppler count interval are given by:

$$\Delta \rho_{1_e} = \frac{1}{10^3 c} \left[\Delta_T \rho(t_{3_e}) + \Delta_{CP} \rho(t_{3_e}) \right] \quad \text{s} \quad (10-24)$$

$$\Delta \rho_{1_s} = \frac{1}{10^3 c} \left[\Delta_T \rho(t_{3_s}) + \Delta_{CP} \rho(t_{3_s}) \right] \quad \text{s} \quad (10-25)$$

where c is the speed of light in kilometers per second, and t_{3_e} and t_{3_s} are reception times in station time ST at the receiving station at the end and start of the doppler count interval.

For the computed value of a one-way wideband spacecraft interferometry (IWS) observable, the media correction to the precision one-way light time at receiving station 2 is calculated from:

$$\Delta\rho_1 = \frac{1}{10^3 c} [\Delta_T \rho(t_3) + \Delta_{CP} \rho(t_3)] \quad \text{s} \quad (10-26)$$

The media correction to the precision one-way light time at receiving station 1 is calculated from the same equation. The reception time t_3 in station time ST is the same at the two stations. However, the troposphere and charged-particle corrections on the paths to the two stations are different.

For the computed value of a one-way narrowband spacecraft interferometry (INS) observable, the media corrections to the precision one-way light times at the end and start of the one-way doppler count interval at receiving station 2 are calculated from Eqs. (10-24) and (10-25). The media corrections to the precision one-way light times at the end and start of the one-way doppler count interval at receiving station 1 are calculated from the same equations. The reception times at the end and start of the count intervals at the two stations are the same, but the media corrections are different.

The observed values of GPS/TOPEX pseudo-range and carrier-phase observables are calculated as a weighted average of values at two different transmitter frequencies, which eliminates the effects of charged particles. Hence, in calculating media corrections for the computed values of these observables, the charged-particle corrections are set to zero. For pseudo-range or carrier-phase observables received at a GPS receiving station on Earth, the media correction to the precision one-way light time is calculated from the first term of Eq. (10-26). For these same observables received at the TOPEX satellite, the media correction is zero.

SECTION 10

10.2.3.2.2 Round-Trip Spacecraft Data Types

For the computed value of a two-way range (ρ_2) or three-way range (ρ_3) observable, the media correction to the precision round-trip light time ρ is calculated from:

$$\Delta\rho = \frac{1}{10^3 c} [\Delta_T\rho(t_3) + \Delta_{CP}\rho(t_3) + \Delta_T\rho(t_1) + \Delta_{CP}\rho(t_1)] \quad \text{s} \quad (10-27)$$

where t_3 and t_1 are the reception and transmission times in station time ST at the receiving and transmitting stations.

For the computed value of a two-way doppler (F_2) or a three-way doppler (F_3) observable, the media corrections to the precision round-trip light times calculated at the end and start of the doppler count interval are calculated from:

$$\Delta\rho_e = \frac{1}{10^3 c} [\Delta_T\rho(t_{3e}) + \Delta_{CP}\rho(t_{3e}) + \Delta_T\rho(t_{1e}) + \Delta_{CP}\rho(t_{1e})] \quad \text{s} \quad (10-28)$$

$$\Delta\rho_s = \frac{1}{10^3 c} [\Delta_T\rho(t_{3s}) + \Delta_{CP}\rho(t_{3s}) + \Delta_T\rho(t_{1s}) + \Delta_{CP}\rho(t_{1s})] \quad \text{s} \quad (10-29)$$

where t_{3e} and t_{3s} are reception times in station time ST at the receiving station at the end and start of the doppler count interval T_c . The epochs t_{1e} and t_{1s} are the corresponding transmission times in station time ST at the transmitting station.

For the computed value of a round-trip wideband spacecraft interferometry (IWS) observable, the media corrections to the precision round-trip light times at receiving stations 2 and 1 should be computed from Eq. (10-27), where the reception time t_3 in station time ST is the same at both stations. However, the two values of the transmission time t_1 differ by a maximum of about 0.02 s and the up-leg media corrections cancel to sufficient accuracy in calculating the media correction to the computed observable. Hence, the up-leg media corrections are not calculated, and the media corrections to the

precision round-trip light times at receiving stations 2 and 1 are calculated from Eq. (10-26) instead of Eq. (10-27). Note that if the up-leg charged-particle corrections were calculated, the sign of this correction would be negative. This occurs because the up leg is a single frequency, and the carrier wave travels at the phase velocity, while the down leg is dual frequency, and the ranging signal travels at the group velocity.

For the computed value of a round-trip narrowband spacecraft interferometry (*INS*) observable, the media corrections to the precision round-trip light times at the end and start of the doppler count interval at receiving stations 2 and 1 should be computed from Eqs. (10-28) and (10-29), where the reception times at the end and start of the count intervals at the two stations are the same. However, for the reasons stated in the preceding paragraph, the up-leg media corrections are not calculated. Hence, the media corrections to the precision round-trip light times at the end and start of the doppler count interval at receiving stations 2 and 1 are calculated from Eqs. (10-24) and (10-25) instead of Eqs. (10-28) and (10-29).

10.2.3.2.3 Quasar Interferometry Data Types

For the computed value of a wideband quasar interferometry (*IWQ*) observable, the media correction to the precision quasar delay τ is calculated from:

$$\Delta\tau = \frac{1}{10^3 c} [\Delta_T \rho(t_2) + \Delta_{CP} \rho(t_2) - \Delta_T \rho(t_1) - \Delta_{CP} \rho(t_1)] \quad \text{s} \quad (10-30)$$

where t_2 and t_1 are the reception times of the quasar wavefront in station time ST at receiving stations 2 and 1, respectively.

For the computed value of a narrowband quasar interferometry (*INQ*) observable, the media corrections to the precision quasar delays at the end and start of the count interval are calculated from:

SECTION 10

$$\Delta \tau_e = \frac{1}{10^3 c} \left[\Delta_T \rho(t_{2e}) + \Delta_{CP} \rho(t_{2e}) - \Delta_T \rho(t_{1e}) - \Delta_{CP} \rho(t_{1e}) \right] \quad \text{s} \quad (10-31)$$

$$\Delta \tau_s = \frac{1}{10^3 c} \left[\Delta_T \rho(t_{2s}) + \Delta_{CP} \rho(t_{2s}) - \Delta_T \rho(t_{1s}) - \Delta_{CP} \rho(t_{1s}) \right] \quad \text{s} \quad (10-32)$$

where t_{1e} and t_{1s} are reception times of the quasar wavefront in station time ST at receiving station 1 at the end and start of the count interval T_c . The epochs t_{2e} and t_{2s} are the corresponding reception times of the quasar wavefront in station time ST at receiving station 2. Note that the count interval T_c is given by Eq. (10-23).

10.2.3.3 Time Arguments for Media Corrections

This section gives approximate expressions for time arguments that are used to calculate tropospheric range corrections and charged-particle range corrections. All parameters in these equations are available from the Regres file. Subsection 10.2.3.3.1 gives equations for the reception time t_3 at the receiving station for spacecraft data types and the reception time t_1 at receiving station 1 for quasar interferometry data types. Subsection 10.2.3.3.2 gives equations for the transmission time t_1 at the transmitting station for spacecraft data types and the reception time t_2 at receiving station 2 for quasar interferometry data types. All calculated reception and transmission times are in station time ST.

10.2.3.3.1 Spacecraft Reception Times, and Quasar Reception Times at Receiving Station 1

For two-way (ρ_2) and three-way (ρ_3) range observables, GPS/TOPEX pseudo-range and carrier-phase observables received at a GPS receiving station on Earth or at the TOPEX satellite, and one-way or round-trip wideband spacecraft interferometry (IWS) observables, the reception time at the receiving station (at each of the two receiving stations for IWS) is the data time tag TT :

$$t_3(\text{ST}) = TT \quad (10-33)$$

For one-way (F_1), two-way (F_2), and three-way (F_3) doppler observables, the reception times at the receiving station at the end and start of the doppler count interval T_c are given by:

$$\begin{aligned} t_{3_e}(\text{ST}) &= TT + \frac{1}{2}T_c \\ t_{3_s}(\text{ST}) &= TT - \frac{1}{2}T_c \end{aligned} \tag{10-34}$$

For one-way and round-trip narrowband spacecraft interferometry (*INS*) observables, the reception times at the end and start of the doppler count interval at each of the two receiving stations are given by Eq. (10-34).

In addition to doppler observables, Regres also calculates the computed values of total-count phase observables. These are doppler observables multiplied by the doppler count interval T_c . Doppler observables are in units of hertz and total-count phase observables are in units of cycles. In addition to the difference in the units of the observables, doppler and total-count phase observables differ in the configuration of the count intervals during a pass of data. Doppler observables have contiguous count intervals of a constant length, such as 60 s, 600 s, or 6000 s. If doppler observables have a count interval of T_c , then the corresponding count intervals for total-count phase observables would be T_c , $2T_c$, $3T_c$, $4T_c$ etc., where all of these count intervals have a common start time near the start of the pass. The last count interval would be almost as long as the pass of data. The time tags for doppler observables are the midpoint of the count interval. For total-count phase observables, the time tag is the end of the count interval. Hence, for total-count phase observables, the reception times at the receiving station at the end and start of the count interval are given by:

$$\begin{aligned} t_{3_e}(\text{ST}) &= TT \\ t_{3_s}(\text{ST}) &= TT - T_c \end{aligned} \tag{10-35}$$

For wideband quasar interferometry (*IWQ*) observables, the reception time at receiving station 1 is the data time tag:

SECTION 10

$$t_1(\text{ST}) = TT \quad (10-36)$$

For narrowband quasar interferometry (*INQ*) observables, the reception times at the end and start of the count interval T_c at receiving station 1 are given by:

$$\begin{aligned} t_{1_e}(\text{ST}) &= TT + \frac{1}{2}T_c \\ t_{1_s}(\text{ST}) &= TT - \frac{1}{2}T_c \end{aligned} \quad (10-37)$$

10.2.3.3.2 Spacecraft Transmission Times, and Quasar Reception Times at Receiving Station 2

For two-way (ρ_2) and three-way (ρ_3) range observables, the transmission time at the transmitting station is given by:

$$t_1(\text{ST}) = TT - \rho \quad (10-38)$$

where ρ is the precision round-trip light time defined by Eq. (10-17). For two-way (F_2) and three-way (F_3) doppler observables, the transmission times at the transmitting station at the end and start of the transmission interval are given by:

$$\begin{aligned} t_{1_e}(\text{ST}) &= TT + \frac{1}{2}T_c - \rho_e \\ t_{1_s}(\text{ST}) &= TT - \frac{1}{2}T_c - \rho_s \end{aligned} \quad (10-39)$$

where ρ_e and ρ_s are precision round-trip light times at the end and start of the doppler count interval, which are defined by Eqs. (10-18) and (10-19).

For two-way (P_2) and three-way (P_3) total-count phase observables, the transmission times at the transmitting station at the end and start of the transmission interval are given by:

$$\begin{aligned} t_{1_e}(\text{ST}) &= TT - \rho_e \\ t_{1_s}(\text{ST}) &= TT - T_c - \rho_s \end{aligned} \tag{10-40}$$

For wideband quasar interferometry (IWQ) observables, the reception time at receiving station 2 is given by:

$$t_2(\text{ST}) = TT + \tau \tag{10-41}$$

where τ is the precision quasar delay defined by Eq. (10-20). For narrowband quasar interferometry (INQ) observables, the reception times at the end and start of the reception interval at receiving station 2 are given by:

$$\begin{aligned} t_{2_e}(\text{ST}) &= TT + \frac{1}{2}T_c + \tau_e \\ t_{2_s}(\text{ST}) &= TT - \frac{1}{2}T_c + \tau_s \end{aligned} \tag{10-42}$$

where τ_e and τ_s are precision quasar delays at the end and start of the count interval, which are defined by Eqs. (10-21) and (10-22).

10.3 IONOSPHERE PARTIALS MODEL

This section gives the ionosphere partials model, which was obtained by differentiating the ionosphere model of Klobuchar (1975) with respect to two parameters of the model. Estimated corrections to these two parameters represent corrections to the charged-particle corrections calculated in the Regres editor (Section 10.2.2). The user can estimate corrections to the two solve-for parameters of the ionosphere which apply at all tracking stations of a specific DSN complex or for a single isolated tracking station. The ionosphere model of Klobuchar is not included in the model for the computed values of observables. Hence, the estimated ionosphere parameters cannot be fed back into the model for computed observables.

Subsection 10.3.1 gives the algorithm for the ionosphere model of Klobuchar and the corresponding partial derivatives with respect to two

SECTION 10

parameters of this model. These individual-leg partials are used in Subsection 10.3.2 to calculate partial derivatives of the three precision light times (ρ_1 , ρ , and τ) with respect to the ionosphere parameters. These same equations are used to calculate partials of the precision light times with respect to the solve-for troposphere parameters (Section 10.2.1.4) and the parameters of the solar corona model (Section 10.4).

10.3.1 IONOSPHERE MODEL AND INDIVIDUAL-LEG PARTIAL DERIVATIVES

Given the unrefracted auxiliary elevation angle γ at the transmitting or receiving station on Earth (Section 9.3), compute the zenith angle Z at the mean ionospheric height of 350 km from:

$$Z = \sin^{-1} (0.94798 \cos \gamma) \quad (10-43)$$

where the numerical coefficient is $a_e / (a_e + 350)$, where a_e is the mean equatorial radius of the Earth (6378.136 km).

Given the auxiliary azimuth angle σ at the transmitting or receiving station on Earth (Section 9.3), calculate the geodetic latitude ϕ_I of the sub-ionospheric point from:

$$\phi_I = \sin^{-1} [\sin \phi_0 \sin(\gamma + Z) + \cos \phi_0 \cos(\gamma + Z) \cos \sigma] \quad (10-44)$$

where ϕ_0 is the geodetic latitude of the transmitting or receiving station on Earth calculated from Eqs. (9-6) to (9-8).

Calculate the east longitude λ_I of the sub-ionospheric point from:

$$\lambda_I = \lambda_0 + \sin^{-1} \left[\frac{\cos(\gamma + Z) \sin \sigma}{\cos \phi_I} \right] \quad (10-45)$$

where λ_0 is the east longitude of the transmitting or receiving station on Earth.

Given the transmission or reception time at the tracking station on Earth in Universal Time UT1 expressed as seconds past January 1, 2000, 12^h UT1, calculate UT1 in hours past the start of the current day from:¹

$$\text{UT1(hours)} = [\text{UT1(seconds)} + 43200, \text{ modulo } 86400] \frac{1}{3600} \quad (10-46)$$

Then calculate the local mean solar time t at the sub-ionospheric point from:

$$t = \text{UT1 (hours)} + \frac{\lambda_I^\circ}{15} \quad \text{hours} \quad (10-47)$$

where λ_I° is λ_I measured in degrees. Add or subtract 24^h to place t in the range of 0 to 24 hours.

The Klobuchar model for the ionospheric range correction in meters is given by:

$$\Delta_I \rho = \pm \frac{1}{\cos Z} (N + D \cos x) \left(\frac{2295 \times 10^6 \text{ Hz}}{f} \right)^2 \quad \text{m} \quad (10-48)$$

where f is the transmitter frequency in Hz for the up leg or the down leg of the light path through the ionosphere. The algorithm for calculating f is given in Section 13.2.8. The sign of the ionospheric range correction $\Delta_I \rho$ given by Eq. (10-48) is negative for doppler and narrowband spacecraft or quasar interferometry data types and positive for range and wideband spacecraft or quasar interferometry data types. The argument x in radians is given by:

¹If Universal Time UT1 is not available, it can be obtained by transforming coordinate time ET as described in Section 5.3.2, steps 1 and 5. However, in the Solar-System barycentric space-time frame of reference, use the approximate expression for ET – TAI. Also, if the resulting UT1 is regularized (*i.e.*, UT1R), it is not necessary to calculate and add the periodic terms ΔUT1 to UT1R to give UT1.

SECTION 10

$$x = \frac{2\pi(t - \phi)}{P} \quad \text{rad} \quad (10-49)$$

where $\phi = 14$ hours and $P = 32$ hours. The coefficient N is the nighttime zenith range correction in meters for a frequency of 2295 MHz. The additional daytime zenith range correction is the positive half of a cosine wave with an amplitude of D meters. This additional term has a peak effect at a local time t of 14 hours. For $|x| = \pi/2$, $|t - \phi| = 8$ hours and the upper half of the cosine wave begins at 6 a.m. and ends at 10 p.m. local time. The term $D \cos x$ is only included if:

$$|x| \leq \frac{\pi}{2} \quad (10-50)$$

The ionospheric range correction at a zenith angle Z is the zenith range correction divided by $\cos Z$.

The parameters of $\Delta_I \rho$ given by Eq. (10-48), which the ODP user can estimate or consider, are the coefficients N and D in meters. The partial derivatives of $\Delta_I \rho$ with respect to these parameters are given by:

$$\frac{\partial \Delta_I \rho}{\partial N} = \pm \frac{1}{\cos Z} \left(\frac{2295 \times 10^6 \text{ Hz}}{f} \right)^2 \quad (10-51)$$

$$\begin{aligned} \frac{\partial \Delta_I \rho}{\partial D} &= \pm \frac{\cos x}{\cos Z} \left(\frac{2295 \times 10^6 \text{ Hz}}{f} \right)^2 & \text{if } |x| \leq \frac{\pi}{2} \\ &= 0 & \text{if } |x| > \frac{\pi}{2} \end{aligned} \quad (10-52)$$

The user can estimate corrections to N and D that apply to all tracking stations of a specific DSN complex or to an isolated tracking station. These parameters can be estimated independently at each DSN complex and at each isolated tracking station.

10.3.2 PARTIAL DERIVATIVES OF PRECISION LIGHT TIMES

From Eqs. (10–26), (10–27), and (10–30), the partial derivatives of the precision one-way light time ρ_1 , the precision round-trip light time ρ , and the precision quasar delay τ with respect to the N and D coefficients of Klobuchar's ionosphere model can be calculated from:

$$\frac{\partial \rho_1}{\partial \mathbf{q}} = \frac{1}{10^3 c} \left[\frac{\partial \Delta_M \rho(t_3)}{\partial \mathbf{q}} \right] \quad (10-53)$$

$$\frac{\partial \rho}{\partial \mathbf{q}} = \frac{1}{10^3 c} \left[\frac{\partial \Delta_M \rho(t_3)}{\partial \mathbf{q}} + \frac{\partial \Delta_M \rho(t_1)}{\partial \mathbf{q}} \right] \quad (10-54)$$

$$\frac{\partial \tau}{\partial \mathbf{q}} = \frac{1}{10^3 c} \left[\frac{\partial \Delta_M \rho(t_2)}{\partial \mathbf{q}} - \frac{\partial \Delta_M \rho(t_1)}{\partial \mathbf{q}} \right] \quad (10-55)$$

where M refers to a particular model and \mathbf{q} is the parameter vector which contains the parameters of that model. For the Klobuchar ionosphere model, $M = I$ which refers to the ionosphere, and the parameter vector \mathbf{q} contains the solve-for coefficients N and D of this model. For spacecraft data types, t_3 is the reception time at the receiving station, and t_1 is the transmission time at the transmitting station. For quasar interferometry data types, t_1 and t_2 are the reception times of the quasar wavefront at receiving stations 1 and 2. For the Klobuchar ionosphere model, the partial derivatives of the five ionospheric range corrections in Eqs. (10–53) to (10–55) with respect to the N and D coefficients of this model are calculated from the formulation given in Section 10.3.1.

Eqs. (10–53) to (10–55) are also used to calculate the partial derivatives of ρ_1 , ρ , and τ with respect to the tropospheric zenith dry and wet range corrections $\Delta \rho_{z_{\text{dry}}}$ and $\Delta \rho_{z_{\text{wet}}}$. For this case, $M = T$ which refers to the troposphere. The individual-leg partials are calculated from the formulation of Section 10.2.1.4.

SECTION 10

The model for solar corona range corrections is given in Section 10.4. Eqs. (10–53) to (10–55) will be used to calculate the partial derivatives of ρ_1 , ρ , and τ with respect to the solve-for parameters A , B , and C of the solar corona model. For this case, $M = SC$ which refers to the solar corona. The formulation for the individual-leg partials is given in Section 10.4.4.

The partial derivatives of ρ_1 , ρ , and τ with respect to the solve-for parameters of the troposphere, ionosphere, and solar corona models are used in Section 12 to calculate the partial derivatives of ρ_1 , ρ , and τ with respect to the solve-for parameter vector \mathbf{q} . These partials, in turn, are used in Section 13 to calculate the partial derivatives of the computed values of each data type with respect to the solve-for parameter vector \mathbf{q} .

10.4 SOLAR CORONA MODEL

This section gives the solar corona model, which was obtained from Anderson (1997) and Muhleman and Anderson (1981). Subsection 10.4.1 gives the formulas for calculating the arguments of the solar corona corrections. The equations for calculating the individual-leg solar corona corrections from these arguments are given in Subsection 10.4.2. The solar corona corrections are calculated within the spacecraft and quasar light-time solutions. Since this was not mentioned in Section 8 (Light-Time Solution), Subsection 10.4.3 indicates how the solar corona range corrections are added to the spacecraft and quasar light-time solutions. Subsection 10.4.4 describes the calculation of the partial derivatives of the individual-leg solar corona range corrections with respect to the A , B , and C coefficients of this model. The equations for calculating the partial derivatives of the precision light times ρ_1 , ρ , and τ with respect to the A , B , and C coefficients of the solar corona model are given in Subsection 10.4.5.

The solar corona corrections and partial derivatives are calculated when program Regres is operating in the Solar-System barycentric space-time frame of reference. They do not apply when calculations are performed in the local geocentric space-time frame of reference.

10.4.1 CALCULATION OF ARGUMENTS FOR SOLAR CORONA CORRECTIONS

Section 10.4.2 gives the formulation for calculating the up-leg or down-leg range correction due to the solar corona. The light-time correction due to the solar corona is the solar corona range correction $\Delta_{SC}\rho$ in meters divided by $10^3 c$, where c is the speed of light in kilometers per second. The solar corona range corrections are a function of the closest approach radius p from the Sun to the light path, the distances from the Sun to the transmitting and receiving stations and the spacecraft, and the latitude ϕ relative to the Sun's equator of the closest approach point to the Sun on the light path. This section gives the formulation for calculating p , the distances to the participants, and ϕ .

A round-trip spacecraft light-time solution in the Solar-System barycentric space-time frame of reference always calculates the Sun-centered space-fixed position vectors of the transmitting station (point 1) at the transmission time t_1 , the spacecraft (point 2) at the reflection time t_2 , and the receiving station (point 3) at the reception time t_3 :

$$\mathbf{r}_3^S(t_3), \mathbf{r}_2^S(t_2), \mathbf{r}_1^S(t_1) \quad (10-56)$$

A quasar light-time solution always calculates the Sun-centered space-fixed position vectors of receiving stations 1 and 2 at the reception times t_1 and t_2 of the quasar wavefront at these stations:

$$\mathbf{r}_2^S(t_2), \mathbf{r}_1^S(t_1) \quad (10-57)$$

Note that transmitting and receiving stations can be tracking stations on Earth or Earth satellites.

In a spacecraft light-time solution, the unit vector \mathbf{L} directed along the Sun-centered light path from the transmitting station at the transmission time t_1 or the receiving station at the reception time t_3 to the spacecraft at the reflection time or transmission time t_2 can be calculated from:

SECTION 10

$$\mathbf{L} = \frac{\mathbf{r}_2^S(t_2) - \mathbf{r}_1^S(t_1)}{|\mathbf{r}_2^S(t_2) - \mathbf{r}_1^S(t_1)|} \quad 1 \rightarrow 3 \quad (10-58)$$

where the bars in the denominator indicate the magnitude of the vector. In a quasar light-time solution, the unit vector \mathbf{L}_Q to the quasar is calculated from Eqs. (8-92) and (8-93).

For the up leg of the light path from a transmitting station to the spacecraft or the down leg of the light path from the spacecraft or a quasar to a receiving station, the position vector \mathbf{p} from the Sun S to the point of closest approach to the Sun on the light path can be calculated from:

$$\mathbf{p} = \mathbf{r}_1^S(t_1) - [\mathbf{r}_1^S(t_1) \cdot \mathbf{L}] \mathbf{L} \quad 1 \rightarrow 2, 3 \quad (10-59)$$

For the up leg of a spacecraft light-time solution, \mathbf{p} is calculated from Eqs. (10-58) and (10-59). For the down leg of a spacecraft light-time solution, \mathbf{p} is calculated from Eqs. (10-58) and (10-59) with subscript 1 changed to 3. For reception of the quasar wavefront at receiving station 1 at the reception time t_1 , calculate \mathbf{p} from Eq. (10-59) with \mathbf{L} replaced with \mathbf{L}_Q . For reception of the quasar wavefront at receiving station 2 at the reception time t_2 , calculate \mathbf{p} from Eq. (10-59) with \mathbf{L} replaced with \mathbf{L}_Q and with subscript 1 changed to 2.

The minimum distance or closest approach radius p from the Sun to the up-leg or down-leg light path is the magnitude of the position vector \mathbf{p} :

$$p = |\mathbf{p}| \quad (10-60)$$

For a spacecraft light-time solution, calculate the distances from the Sun to the transmitting station at t_1 , the spacecraft at t_2 , and the receiving station at t_3 as the magnitudes of the position vectors given in (10-56):

$$r_1 = |\mathbf{r}_1^S(t_1)| \quad 1 \rightarrow 2, 3 \quad (10-61)$$

For a quasar light-time solution, use this equation to calculate the distances from the Sun to receiving station 1 at t_1 and receiving station 2 at t_2 as the magnitudes of the position vectors given in (10–57).

The unit vector \mathbf{P} directed toward the Sun's mean north pole (axis of rotation) of date is calculated from:

$$\mathbf{P} = \begin{bmatrix} \cos \delta \cos \alpha \\ \cos \delta \sin \alpha \\ \sin \delta \end{bmatrix} \quad (10-62)$$

where the right ascension α and declination δ of the Sun's mean north pole of date relative to the mean Earth equator and equinox of J2000 are calculated from Eqs. (6–8) and (6–9) using inputs obtained from the GIN file. These angles are currently constants. However, when rate terms are added, these angles can be calculated to sufficient accuracy from the transmission time t_1 and reception time t_3 for the up and down legs of the spacecraft light path and the reception times t_1 and t_2 for the down legs of the quasar light path, where all epochs are in coordinate time ET. Using \mathbf{p} and p from Eqs. (10–59) and (10–60) and \mathbf{P} from Eq. (10–62), the latitude ϕ relative to the Sun's mean equator of date of the closest approach point to the Sun on the up-leg or down-leg light path can be calculated from:

$$\phi = \sin^{-1} \left[\mathbf{P} \cdot \frac{\mathbf{p}}{p} \right] \quad (10-63)$$

and converted to degrees.

10.4.2 INDIVIDUAL-LEG SOLAR CORONA CORRECTIONS

The up-leg and down-leg solar corona range corrections are calculated from Eq. (1) of Anderson (1997), which is supported by the theory of Muhleman and Anderson (1981):

SECTION 10

$$\Delta_{SC}\rho = \pm \left[A \left(\frac{R_S}{p} \right)^F + B \left(\frac{R_S}{p} \right)^{1.7} e^{-\left(\frac{\phi}{\phi_0} \right)^2} + C \left(\frac{R_S}{p} \right)^5 \right] \left(\frac{2295 \times 10^6 \text{ Hz}}{f} \right)^2 \quad \text{m} \quad (10-64)$$

where

$$\begin{aligned} A, B, C &= \text{solve-for parameters, m} \\ R_S &= \text{radius of Sun} = 696,000 \text{ km} \\ \phi_0 &= \text{reference latitude} = 10^\circ \\ f &= \text{up-leg or down-leg carrier frequency, Hz} \end{aligned}$$

The closest approach radius p from the Sun and the latitude ϕ relative to the Sun's mean equator of date of the closest approach point to the Sun are calculated from equations given in Section 10.4.1. For a spacecraft light-time solution, the factor F in Eq. (10-64) is calculated from Eq. (8) of Anderson (1997):

$$F = \frac{1}{\pi} \tan^{-1} \frac{\sqrt{r_2^2 - p^2}}{p} + \frac{1}{\pi} \tan^{-1} \frac{\sqrt{r_1^2 - p^2}}{p} \quad 1 \rightarrow 3 \quad (10-65)$$

which applies for the up leg of the light path. For the down leg, change the subscript 1 to 3. The radii r_1 , r_2 , and r_3 from the Sun to the transmitting station, spacecraft, and receiving station are calculated from Eq. (10-61). For a quasar light-time solution, on the down-leg light path to receiving station 1, F is calculated from:

$$F = \frac{1}{2} + \frac{1}{\pi} \tan^{-1} \frac{\sqrt{r_1^2 - p^2}}{p} \quad 1 \rightarrow 2 \quad (10-66)$$

For the down-leg light path to receiving station 2, change the subscript 1 to 2. The radii r_1 and r_2 from the Sun to receiving stations 1 and 2 are calculated from Eq. (10-61).

The algorithm for calculating the transmitter frequency f for each leg of each light path is given in Section 13.2.8.

The sign of the solar corona range correction $\Delta_{SC\rho}$ given by Eq. (10–64) is negative for doppler and narrowband spacecraft or quasar interferometry data types and positive for range and wideband spacecraft or quasar interferometry data types. GPS/TOPEX carrier-phase and pseudo-range observables are calculated as a weighted average, which eliminates the effects of charged particles (see Section 11.5). Hence, solar corona range corrections are not calculated for these data types.

The individual-leg solar corona range corrections calculated from Eqs. (10–64) to (10–66) are only valid when the transmitting and receiving stations (on Earth or in Earth orbit) are on one side of the Sun and the spacecraft or the quasar is on the opposite side of the Sun. The following equations can be used to determine when this geometry occurs.

For the up leg of the light path from the transmitting station to the spacecraft or the down leg of the light path from the spacecraft to the receiving station, compute the solar corona range correction if the following two inequalities are satisfied:

$$\mathbf{r}_1^S(t_1) \cdot \mathbf{L} < 0 \quad 1 \rightarrow 3 \quad (10-67)$$

$$\mathbf{r}_2^S(t_2) \cdot \mathbf{L} > 0 \quad (10-68)$$

where \mathbf{L} is given by Eq. (10–58).

For the down-leg light path from the quasar to the receiving station, calculate the solar corona range correction if the following inequality is satisfied:

$$\mathbf{r}_1^S(t_1) \cdot \mathbf{L}_Q < 0 \quad 1 \rightarrow 2 \quad (10-69)$$

where \mathbf{L}_Q is calculated from Eqs. (8–92) and (8–93).

SECTION 10

In calculating the computed value of an observable, one, two, or four light-time solutions are obtained, each of which has one or two legs. For each computed observable, calculate solar corona range corrections for all legs of all light-time solutions or for none of them. The easiest way to accomplish this is to apply the above inequalities to the first leg of the first light-time solution calculated. If a solar corona range correction is calculated for the first leg, then calculate solar corona range corrections for all remaining legs for that data point. Conversely, if a solar corona range correction is not calculated for the first leg, then do not calculate solar corona range corrections for any of the remaining legs for that data point.

10.4.3 ADDING SOLAR CORONA CORRECTIONS TO THE LIGHT-TIME SOLUTIONS

The solar corona range corrections are calculated from the formulas given in Sections 10.4.1 and 10.4.2.

For a spacecraft light-time solution, the down-leg solar corona range correction must be calculated in Step 14 of the spacecraft light-time solution (Section 8.3.6), along with all of the other quantities computed on the down-leg light path. The up-leg solar corona range correction must be calculated along with the other quantities computed on the up-leg light path in Step 27. Note that the solar corona range corrections are only calculated in the Solar-System barycentric space-time frame of reference.

Eq. (8-72) is used to differentially correct the transmission time $t_2(\text{ET})$ for the down leg of the spacecraft light-time solution, and the transmission time $t_1(\text{ET})$ for the up leg. The negative of the light-time correction due to the solar corona must be added to the numerator of this equation. That is, add the following term to the numerator of Eq. (8-72):

$$-\frac{\Delta_{\text{SC}}\rho_{ij}}{10^3 c} \quad (10-70)$$

When Eq. (8-72) is applied to the down leg of the spacecraft light-time solution, $i = 2$ and $j = 3$, and $\Delta_{SC}\rho_{23}$ is the down-leg solar corona range correction in meters. When Eq. (8-72) is applied to the up leg of the spacecraft light-time solution, $i = 1$ and $j = 2$, and $\Delta_{SC}\rho_{12}$ is the up-leg solar corona range correction. Use the modified form of Eq. (8-72) to differentially correct the transmission time $t_2(\text{ET})$ for the down leg of the light path in Step 15 of the spacecraft light-time solution and to differentially correct the transmission time $t_1(\text{ET})$ for the up leg of the light path in Step 28.

For a quasar light-time solution, the down-leg solar corona range correction $\Delta_{SC}\rho_2$ for receiving station 2 with reception time $t_2(\text{ET})$ and the down-leg solar corona range correction $\Delta_{SC}\rho_1$ for receiving station 1 with reception time $t_1(\text{ET})$ must be calculated in Step 10 of the quasar light-time solution (Section 8.4.3) along with all of the other quantities which are associated with the travel time of the quasar wavefront from receiving station 1 to receiving station 2.

In a quasar light-time solution, Eq. (8-103) is used to differentially correct the reception time $t_2(\text{ET})$ of the quasar wavefront at receiving station 2. The negative of the correction to the light time $t_2(\text{ET}) - t_1(\text{ET})$ due to the solar corona must be added to the numerator of this equation. That is, add the following term to the numerator of Eq. (8-103):

$$-\frac{[\Delta_{SC}\rho_2 - \Delta_{SC}\rho_1]}{10^3 c} \quad (10-71)$$

Use the modified form of Eq. (8-103) to differentially correct the reception time $t_2(\text{ET})$ of the quasar wavefront at receiving station 2 in Step 11 of the quasar light-time solution.

10.4.4 INDIVIDUAL-LEG SOLAR CORONA PARTIAL DERIVATIVES

From Eq. (10-64), the partial derivatives of the solar corona range correction $\Delta_{SC}\rho$ with respect to the solve-for parameters A , B , and C of the solar corona model are given by:

$$\frac{\partial \Delta_{SC} \rho}{\partial A} = \pm \left(\frac{R_S}{p} \right) F \left(\frac{2295 \times 10^6 \text{ Hz}}{f} \right)^2 \quad (10-72)$$

$$\frac{\partial \Delta_{SC} \rho}{\partial B} = \pm \left(\frac{R_S}{p} \right)^{1.7} e^{-\left(\frac{\phi}{\phi_0} \right)^2} \left(\frac{2295 \times 10^6 \text{ Hz}}{f} \right)^2 \quad (10-73)$$

$$\frac{\partial \Delta_{SC} \rho}{\partial C} = \pm \left(\frac{R_S}{p} \right)^5 \left(\frac{2295 \times 10^6 \text{ Hz}}{f} \right)^2 \quad (10-74)$$

where all quantities in these equations are calculated from the formulas in Sections 10.4.1 and 10.4.2.

10.4.5 PARTIAL DERIVATIVES OF PRECISION LIGHT TIMES

The partial derivatives of the precision one-way light time ρ_1 , the precision round-trip light time ρ , and the precision quasar delay τ with respect to the solve-for A , B , and C coefficients of the solar corona model are given by Eqs. (10-53) to (10-55), which are rewritten here with a slight change of notation:

$$\frac{\partial \rho_1}{\partial \mathbf{q}} = \frac{1}{10^3 c} \left[\frac{\partial \Delta_{SC} \rho_{23}}{\partial \mathbf{q}} \right] \quad (10-75)$$

$$\frac{\partial \rho}{\partial \mathbf{q}} = \frac{1}{10^3 c} \left[\frac{\partial \Delta_{SC} \rho_{23}}{\partial \mathbf{q}} + \frac{\partial \Delta_{SC} \rho_{12}}{\partial \mathbf{q}} \right] \quad (10-76)$$

$$\frac{\partial \tau}{\partial \mathbf{q}} = \frac{1}{10^3 c} \left[\frac{\partial \Delta_{SC} \rho_2}{\partial \mathbf{q}} - \frac{\partial \Delta_{SC} \rho_1}{\partial \mathbf{q}} \right] \quad (10-77)$$

where the subscripts on the solar corona range corrections are defined in Section 10.4.3. The parameter vector \mathbf{q} contains the solve-for parameters A , B , and C of the solar corona model. The partial derivatives of the five solar corona range corrections in Eqs. (10-75) to (10-77) with respect to the A , B , and C coefficients are calculated from Eqs. (10-72) to (10-74).

The partial derivatives of ρ_1 , ρ , and τ with respect to the solve-for parameters of the solar corona model are used in Section 12 to calculate the partial derivatives of ρ_1 , ρ , and τ with respect to the solve-for parameter vector \mathbf{q} . These partials, in turn, are used in Section 13 to calculate the partial derivatives of the computed values of each data type with respect to the solve-for parameter vector \mathbf{q} .

10.5 ANTENNA CORRECTIONS

10.5.1 INTRODUCTION

Antenna corrections are calculated at the reception time at a receiving station on Earth and at the transmission time at a transmitting station on Earth. These calculated corrections are non-zero if the primary and secondary axes of the antenna do not intersect. The axis-offset b is the perpendicular distance in meters between the centerlines of the primary and secondary axes of the antenna. As the antenna rotates to track a spacecraft or a quasar, the primary axis rotates and the secondary axis moves relative to the Earth if the axis-offset b is non-zero. If the axis-offset b is non-zero for an antenna, the antenna correction for that antenna is non zero.

The light time (calculated in the light-time solution) from the transmitting station on Earth to the spacecraft and from the spacecraft to the receiving station on Earth is based upon transmission and reception at the station location, which is on the primary axis of the antenna, where the secondary axis would intersect it if the axis-offset b were reduced to zero. On the other hand, range observables, which measure the round-trip light time to the spacecraft, are calibrated for transmission and reception at the secondary axis of the antenna. The antenna corrections change the calculated light times from transmission and reception on the primary axis of the antenna (two antennas if the transmitting station is not the receiving station) to transmission and reception on the secondary axis of the antenna (or antennas).

For the up leg of the light path from a transmitting station on Earth to the spacecraft, the antenna correction in seconds is the negative of the travel time of

SECTION 10

the transmitted wavefront from its intersection with the primary axis of the antenna to its intersection with the secondary axis of the antenna. Similarly, for the down leg of the light path from the spacecraft or a quasar to a receiving station on Earth, the antenna correction in seconds is the negative of the travel time of the received wavefront from its intersection with the secondary axis of the antenna to its intersection with the primary axis of the antenna. The antenna correction in seconds is the antenna correction $\Delta_A \rho$ in meters divided by $10^3 c$, where c is the speed of light in kilometers per second. The antenna correction in meters has the general form:

$$\Delta_A \rho(t_i) = -b \cos \theta(t_i) \quad i = 1, 2, 3 \quad \text{m} \quad (10-78)$$

where b is the axis-offset in meters and θ is the secondary angle of the antenna at the transmission time or reception time t_i . Antenna corrections are calculated at the transmission time t_1 and reception time t_3 for round-trip spacecraft light-time solutions and at the reception times t_1 and t_2 at receiving stations 1 and 2 for quasar light-time solutions. Referring to Figures 9-1, 9-3, 9-4, and 9-5, the secondary angles for HA-DEC (angle pair $HA-\delta$), AZ-EL (angle pair $\sigma-\gamma$), $X-Y$, and $X'-Y'$ antennas are the angles declination δ , elevation γ , Y , and Y' , respectively. Antenna corrections calculated at the transmission or reception time t_i are calculated from unrefracted auxiliary angles (Section 9), which are calculated at that time. The antenna corrections should be calculated from refracted angles. However, the errors in the computed values of observables due to calculating antenna corrections from unrefracted auxiliary angles are negligible.

Figure 10-1 shows the antenna geometry for any antenna at the transmission time or reception time at a tracking station on Earth. The primary axis of the antenna is in the plane of the paper and the secondary axis, which is offset from the primary axis by the axis-offset b , is perpendicular to the plane of the paper. The angle θ is the secondary angle of the antenna (*e.g.*, the elevation angle γ for an AZ-EL mount antenna). The paths from the antenna to the spacecraft (or quasar) and from the station location on the primary axis to the

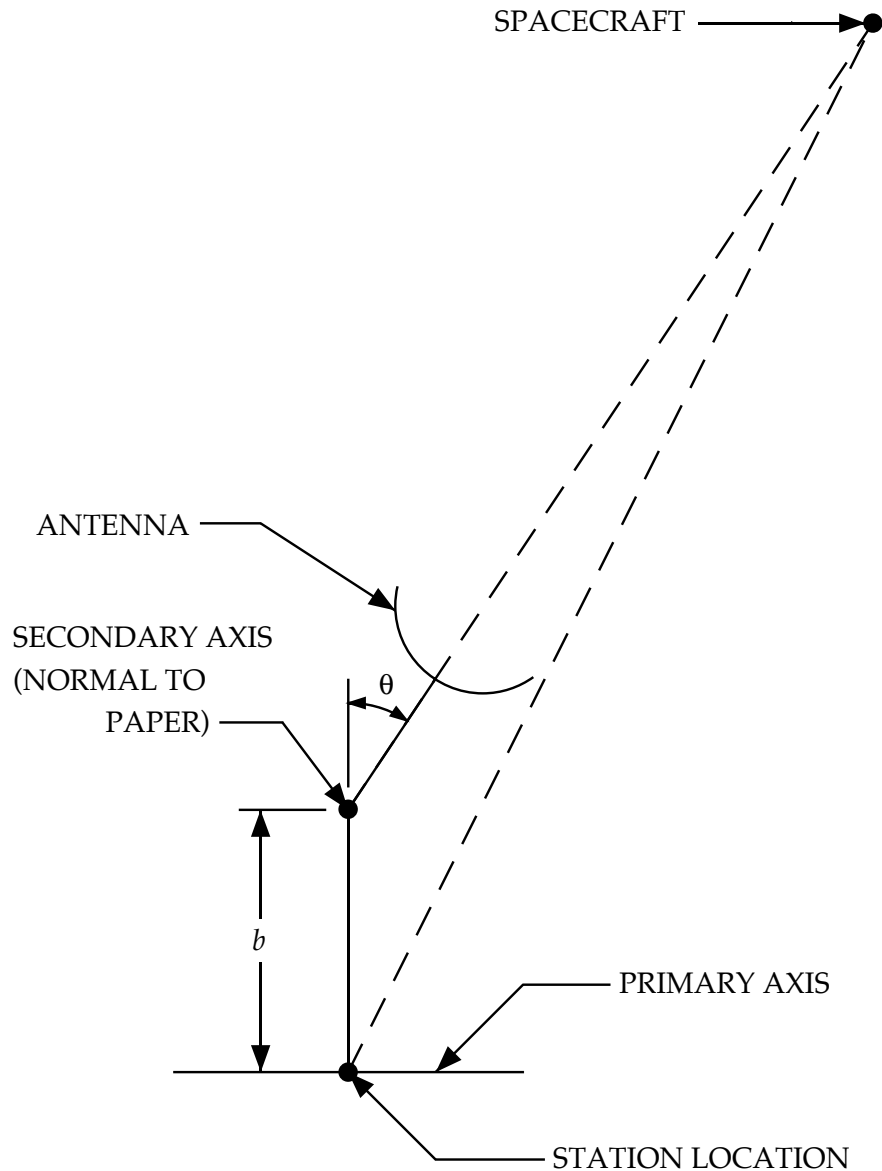


Figure 10-1 Antenna Correction

spacecraft (or quasar) become parallel as the spacecraft recedes to infinity. It is clear that the distance from the station location to the spacecraft is larger than the distance from the secondary axis (the tracking point) to the spacecraft by $b \cos \theta$. Since the calculated light time between the station location and the spacecraft or quasar (in the spacecraft or quasar light-time solution) is corrected with the additive antenna correction so that it will be equal to the observed light time

SECTION 10

between the secondary axis (the observed light time is calibrated to this point) and the spacecraft or quasar, the antenna correction $\Delta_A\rho$ in meters is the negative of $b \cos\theta$, and the corresponding light-time correction is $\Delta_A\rho$ in meters given by Eq. (10–78) divided by 10^3c , where c is the speed of light in kilometers per second.

The calculated antenna corrections given by Eq. (10–78) divided by 10^3c are included in the expressions given in Section 11 for calculating the precision one-way light time ρ_1 , the precision round-trip light time ρ , and the precision quasar delay τ . The expression for ρ_1 includes the antenna correction in seconds at the reception time t_3 . The expression for ρ includes antenna corrections at the reception time t_3 and at the transmission time t_1 . The expression for τ includes the antenna correction at the reception time t_2 at receiving station 2 and the negative of the antenna correction at the reception time t_1 at receiving station 1.

10.5.2 ANTENNA TYPES AND CORRECTIONS

This section describes the types of antennas that exist at the tracking stations of the Deep Space Network (DSN), the axis-offsets b , and the equations used to calculate the antenna corrections $\Delta_A\rho$ in meters. It also gives a six-digit antenna identifier that describes each size and type of antenna. If the antenna identifier for a specific tracking station matches one of the five identifiers for which antenna corrections are computed, then that type of antenna correction is calculated for that tracking station. Otherwise, the antenna correction for that station is zero.

Table 10–1 summarizes the types of antennas which exist at the tracking stations of the Deep Space Network (DSN). Column 1 gives the antenna diameter in meters. Column 2 lists the angle pair measured by the antenna. The antenna identifier is listed in column 3. The axis-offset b in meters is given in column 4. The secondary angle of the antenna in the notation of the computed auxiliary angles is given in column 5.

Table 10–1
Antenna Types

Antenna Diameter m	Angle Pair	Antenna Identifier	Axis-Offset b m	Secondary Angle
26 or 34	HA-DEC	26-H-D 34-H-D	6.706	δ
26	AZ-EL	26-A-E	0.9144	γ
26	X'-Y'	26-X-Y	6.706	Y'
9	X-Y	9-X-Y	2.438	Y
34	AZ-EL	34-HSB	1.8288	γ
34	AZ-EL	34-HEF	0	γ
34	AZ-EL	34-BWG	0	γ
64 or 70	AZ-EL	64-A-E 70-A-E	0	γ
11	AZ-EL	11VLBI	0	γ

For 26-m or 34-m hour angle-declination (HA-DEC) antennas, the antenna type is specified as 26-H-D or 34-H-D and program Regres calculates the antenna correction in meters from:

$$\Delta_A \rho = -b \cos \delta \quad \text{m} \quad (10-79)$$

where δ is the declination angle of the spacecraft or quasar. The axis-offset b is 6.706 m.

SECTION 10

For the 26-m azimuth-elevation (AZ-EL) antenna, the antenna type is specified as 26-A-E and the antenna correction is calculated from:

$$\Delta_A \rho = -b \cos \gamma \quad \text{m} \quad (10-80)$$

where γ is the elevation angle of the spacecraft or quasar. The axis-offset b is 0.9144 m.

For 26-m X-Y mount antennas, the antenna type is specified as 26-X-Y and the antenna correction is calculated from:

$$\Delta_A \rho = -b \cos Y' \quad \text{m} \quad (10-81)$$

where the secondary angle of the antenna is the auxiliary angle Y' . Note that the 26-m X-Y mount antennas actually measure the angles X' and Y' as shown in Figure 9-5. The axis-offset b is 6.706 m.

For 9-m X-Y mount antennas, the antenna type is specified as 9-X-Y, and the antenna correction is calculated from:

$$\Delta_A \rho = -b \cos Y \quad \text{m} \quad (10-82)$$

where the secondary angle of the antenna is the auxiliary angle Y . The 9-m X-Y mount antennas measure the angles X and Y as shown in Figure 9-4. The axis offset b is 2.438 m.

For 34-m AZ-EL mount high-speed beam wave guide antennas, the antenna type is specified as 34-HSB, the axis-offset b is 6 feet or 1.8288 m, and the antenna correction can be calculated from Eq. (10-80).

For 34-m AZ-EL mount high efficiency antennas, the azimuth and elevation axes intersect and the antenna correction is zero. This antenna type is specified as 34-HEF.

For 34-m AZ-EL mount beam wave guide antennas, the axes intersect and the antenna correction is zero. This antenna type is specified as 34-BWG .

For 64-m or 70-m AZ-EL mount antennas, the axes intersect and the antenna correction is zero. These antennas can be specified as 64-A-E or 70-A-E.

For 11-m AZ-EL mount orbiting VLBI antennas, the azimuth and elevation axes intersect and the antenna correction is zero. These antennas are specified as 11VLBI. The upper part of the azimuth axis of the antenna is mounted on a wedge which tips the azimuth axis away from the vertical by 7° . The lower surface of the wedge is horizontal, and the upper surface is tipped 7° from the horizontal. The wedge can be rotated about a vertical axis. Let

σ = azimuth angle in degrees east of north of the high point
of the wedge

The azimuth axis is tipped away from the vertical by 7° in the direction $\sigma + 180^\circ$ east of north. The azimuth of the wedge σ (the so-called train angle) is held fixed during a pass of the spacecraft over the antenna. The station location is the intersection of the tipped azimuth axis and the elevation axis. It is located on the circumference of a horizontal circle of radius r . The current estimate of this radius is:

$$r = 39.838 \text{ cm} = 0.39838 \times 10^{-3} \text{ km} \quad (10-83)$$

The solve-for station location is the center of the horizontal circle. The Earth-fixed vector from the solve-for station location to the actual station location is given by:

$$\Delta \mathbf{r}_b = -r \cos \sigma \mathbf{N} - r \sin \sigma \mathbf{E} \quad \text{km} \quad (10-84)$$

where the components of the vectors are referred to the Earth-fixed coordinate system aligned with the true pole, prime meridian, and equator of date. The north and east vectors are calculated from Eqs. (9-3) to (9-8). For 11-m AZ-EL mount orbiting VLBI antennas, the station location offset vector $\Delta \mathbf{r}_b$ given by Eq. (10-84) is added to the Earth-fixed position vector \mathbf{r}_b of the tracking station given by Eq. (5-1).